PD – 090

II Year M.Sc. (DCC) Degree Examination, January 2018
(Fresh and Repeaters) (Y2K13 Scheme)
MATHEMATICS
M202 : Numerical Analysis

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **any five** questions, choosing at least **two** from **each** Part.*

*2) **All** questions carry **equal** marks.*

PART – A

1. a) Find the smallest root of the equation $f(x) = 3x - \cos x - 1 = 0$ correct to four decimals.

   b) Determine a quadratic factor of the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$ with $p_0 = 0.5 = q_0$. **(8+8)**

2. a) Explain the procedure of Gauss-elimination method for solving the system of equations and hence solve

   $2x_1 + 2x_2 + x_3 + 2x_4 = 7$

   $x_1 - 2x_2 - x_4 = 2$

   $3x_1 - x_2 - 2x_3 - x_4 = 3$

   $x_1 - 2x_4 = 0$.

   b) Find the roots of

   $x^2 - y^2 = 4$

   $x^2 + y^2 = 16$

   with $x_0 = y_0 = 2\sqrt{2}$. **(8+8)**

P.T.O.

3. a) With help of the following table :

x	: –1	0	1
f(x)	: 1	1	3
f′(x)	: –5	1	7

Find f(–0.5) and f(0.5) using the Hermite interpolation.

b) Obtain the Rational approximation $R_{5,4}$ for e^{-x}. **(8+8)**

4. a) Derive the Gauss-Legendre three point integration formula and hence evaluate $\int_{-1}^{1}(1-x^2)^{3/2}\cos x\,dx$.

b) Evaluate : $\int_0^1\int_0^x 4xy\,dydx$ using Simpson's $1/3$ rule with three subintervals. **(8+8)**

PART – B

5. a) Derive the Runge-Kutta fourth order method and hence solve $\frac{dy}{dx}=\frac{x^2}{y^2+1}$, y(0) = 0 at x = 0.2. Take h = 0.1.

b) Using the Runge-Kutta second order method, solve

$$\frac{dy}{dx} = -3y + 2z,\ y(0) = 0$$

$$\frac{dz}{dx} = 3y - 4z,\ z(0) = 0.5$$

with h = 0.2. Obtain the solution at x = 0.4. **(8+8)**

6. a) Solve the boundary value problem $y'' = xy$, $y(0) = y'(0) = 1$, $y(1) = 1$ with $\Delta x = \frac{1}{3}$ using finite difference method.

b) Solve the one-dimensional equation $U_t = U_{xx}, 0 \le x \le 1, t \ge 0$ with

$U(x, 0) = \sin 2\pi x; 0 \le x \le 1$

$U(0, t) = 0 = U(1, t), t \ge 0$

using Crank-Nicolson method.

Take $\Delta x = \frac{1}{4}, \Delta k = \frac{1}{36}$. Obtain the solution at second time level. **(8+8)**

7. Derive the alternating direction implicit scheme applied to the two dimensional parabolic equation. Also discuss its stability issues. **16**

8. a) Discuss the stability of the finite difference scheme applied to the one-dimensional wave equation.

b) Solve the Poisson's equation $U_{xx} + U_{yy} = \sin \pi x \sin \pi y$, $0 \le x, y \le 1$ subjected to the conditions $U = 0$ on the boundary. Take $\Delta x = \Delta y = \frac{1}{3}$. **(6+10)**